CSE 301: Mathematical Analysis for Computer Science
Class Test 2 (Integer Functions and Number Theory)

Name: Student No.

Write in the left margin "T/F" for True/False.

1. $\lceil -x \rceil = -\lfloor x \rfloor$ T
2. $\lfloor x - n \rfloor = \lfloor x \rfloor - n$ T
3. $\lceil x \rceil = n \Leftrightarrow x\text{-}1 < x \leq n$ T
4. $n \leq x \Leftrightarrow n \leq \lfloor x \rfloor$ T
5. $\lceil \sqrt{\lceil x \rceil} \rceil = \lceil \sqrt{x} \rceil$ T
6. -5 mod -1 = 0 T
7. If $n = 2^m + l$, where $0 \leq l < 2^m$, then $l = n - 2^{\lfloor \log n \rfloor}$. T
8. Suppose that **round**(x) means the nearest integer of x; if x is n.5, for some integer n, then it is rounded **up**. Then, **round**(x) = $\lceil x + .5 \rceil$. F
9. For an integer n and a real x, we know that $\lfloor nx \rfloor = \lfloor n\lfloor x \rfloor + n\{x\} \rfloor = n\lfloor x \rfloor + \lfloor n\{x\} \rfloor$. Therefore, $\lfloor nx \rfloor = n\lfloor x \rfloor \Leftrightarrow \lfloor n\{x\} \rfloor = 0 \Leftrightarrow 0 \leq n\{x\} < 1$. T
10. Let the m be the n-th number in the sequence: 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, …. Then, $m(m - 1)/2 < n \leq m(m + 1)/2$. T
11. $k \backslash m$ and $k \backslash n \Leftrightarrow k \backslash \gcd(m, n)$ T
12. 30031 is prime.
    F. 30031 = 59*509
13. Since it is possible to prove that any Euclid number e_n **mod** 5 = 2 or 3, the statement "every prime number is a factor of some Euclid number" is false.
    T. 5 is a prime, but it is not factor of any Euclid number.
14. 100! is divisible by 2^{197}. F
15. m and n are relatively prime means at least one of them is prime. F
16. $ad \equiv d^2 \Leftrightarrow a \equiv d$ F
17. Testing whether a number n is prime or not can be done in $O(\log n)$ time.
    T. By AKS algorithm.
18. $n! = (2\pi n)^{1/2}(n/e)^n$
    F. It should be $\approx$.
19. If p is prime, then $a^{p-1} \equiv 1 \pmod{p}$ for all integer a.
    F. It should be $a^p \equiv a$, and unless a and p are not relatively prime, from here we can not write $a^{p-1} \equiv 1 \pmod{p}$.
20. If n is not prime, then it is always possible to find an integer $x < n$ such that $x^{n-1} \bmod n \neq 1$.
    T