CSE 301: Mathematical Analysis for Computer Science
Class Test 3 (Binomial Coefficient and Special Numbers)

Name: Student No.

Write in the left margin "T/F" for True/False.

1. $\binom{r}{k} = \frac{r}{k}\binom{r/2}{k/2}$ F

2. $\binom{r}{k} = \binom{r-1}{k} + \binom{r-1}{k-1}$, because choosing k items from r items is equivalent to always excluding one specific item and choosing k items from the remaining r-1 items or always including the specific item and choosing k-1 items from the remaining r-1 items. T

3. $\sum_{k=1}^{n}\binom{n}{k} = 2^n$ F

4. $\left\{ {n \atop 0} \right\} = 0$, for all $n \geq 0$ F

5. $\left[{n \atop 0} \right] = 0$, for all $n \geq 0$ F

6. $\left\{ {n \atop 1} \right\} = 1$, for all $n \geq 1$ T

7. $\left[{n \atop 1} \right] = 1$, for all $n \geq 1$ F

8. $\left\{ {n \atop 2} \right\} = 2^n - 1$ F

9. $\left[{n \atop k} \right] \geq \left\{ {n \atop k} \right\}$, for $n,k \geq 0$ T

10. Given a clockwise cycle of n distinct elements, a new element can be inserted into it in n-1 different ways. F (n)

11. $\left\{ {n \atop k} \right\} = k\left\{ {n-1 \atop k} \right\} + \left\{ {n-1 \atop k-1} \right\}$ T

12. $\left[{n \atop k} \right] = (k-1)\left[{n-1 \atop k-1} \right] + \left[{n-1 \atop k} \right]$ F

13. In each combination of $\left[{n-1 \atop k} \right]$, inserting a new element over all k cycles makes (n-1) new cycles. T

14. $\left\langle {n \atop 0} \right\rangle = 1$, for all $n \geq 0$ T

15. $\left\langle {n \atop n} \right\rangle = 0$, for all $n \geq 1$ T

16. $\left\langle {n \atop k} \right\rangle = \left\langle {n \atop n-1-k} \right\rangle$ T

17. Inserting a new element in a given permutation can increase the number of ascent by at most one. T

18. Total number of ascents and descents in an n-element permutation is always n-1. T

19. $1 + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{2n-1} = H_{2n-1} - H_{n-1}$ F

20. $\sum_{k=0}^{n} F_k = F_{n+2} - 1$
    T. Add F_1 in both side and then gradually squeeze the left side to larger Fibonacci numbers.