CSE 421N: Basic Graph Theory
Class Test 2, 07/09/2008
Full Marks: 20, Time:20 mins

Student No. Name:

Write "T" for true and "F" for false for each of the following. Write in the left margin.

1. Every 2-connected graph is also 3-connected.
   [F: The reverse is true]
2. A k-connected graph has at least k vertices.
   [T: Otherwise "k-connected"-ness is meaningless]
3. A block is 2-connected but not 3-connected.
   [F: A block can be k-connected for any value of $k \geq 1$.]
4. A single vertex can not be a block.
   [F: An isolated vertex is a block]
5. If a graph is not 3-regular then its vertex connectivity and edge connectivity can not be the same.
   [F: For a cycle, vertex and edge connectivity are the same, but it is not 3-regular]
6. For a 2-regular graph, $k = k' = 2$.
   [F: For a disconnected 2-regular graph (two disjoint cycles) $k = k' = 0$]
7. In a block-cutpoint graph, there is an edge between two vertices when their corresponding blocks share a vertex.
   [F: In a block-cutpoint graph, an edge is in between a block and a cutpoint]
8. The block-cutpoint graph of a connected graph is connected.
   [T: We know that it is a tree, so it must be connected]
9. Suppose G is a k-connected graph. If a new vertex x is added by connecting k+1 edges to G, then the resulting graph becomes (k+1)-connected.
   [F: In the resulting graph, sub graph G is still k-connected, so it can be possible to make that G disconnected by removing k vertices]
10. Suppose G is a k-connected graph. If a new vertex x is added by connecting k-1 edges to G, then the resulting graph becomes (k-1)-connected.
    [T: Remove the (k-1) vertices to which x have become adjacent. It will disconnect x.]
11. Every k-edge connected graph is k-connected.
    [F: A bowtie (page 12) is 2-edge connected. But it is 1-connected and thus not 2-connected]
12. Bipartite graphs are 2-connected.
    [F: A bipartite graph can even be disconnected]
13. 3-regular bipartite graphs are 3-connected.
    [T:
14. Any two vertices in a 2-connected graph have two vertex disjoint, but not necessarily edge disjoint, paths.
    [F: Vertex disjoint implies edge disjoint]
15. If any two vertices of a graph G have two vertex disjoint paths, then G is 2-connected.
    [T: This is another definition of connectivity]
16. Suppose G is connected. After subdivision of an edge e of G, the graph remains connected.
    [T: No explanation required]
17. An n-vertex graph is not (n+1)-connected.
    [T: In an n-vertex graph, at most (n-1) vertices can be deleted. So it can be at most (n-1)-connected]
18. Any two vertices in a k-connected graph have exactly k vertex-disjoint paths.
    [F: **At least** k such paths]
19. Menger's theorem: A graph is 2-connected iff it has an ear decomposition.
    [F: This is not Menger's theorem, see the book]
20. Each vertex of an ear has degree two.
    [F: The two end points have degree at least three, internal vertices have degree exactly two]