CSE 421N: Basic Graph Theory
Class Test 3, 14/10/2008
Full Marks: 20, Time:20 mins

Student No. Name:

Write “T” for true and “F” for false for each of the following. Write in the left margin.

1. Only bipartite graphs are 2-colorable.
   [T: A graph is 2-colorable **iff** it is bipartite]
2. Petersen graph is 2-colorable.
   [F: Petersen graph is not bipartite, so not 2-colorable; It is tri-partite, so 3-colorable]
3. $\chi \leq \omega$.
   [F: $\chi \geq \omega$]
4. $\chi \geq n/\omega$.
   [F: A path of much length, say 99, has $n = 100$ and $\omega = 2$, but $\chi = 2$ only]
5. $\chi \leq \Delta + 1$.
   [T: Greedy Coloring gives this bound]
6. The graph coloring problem is NP-complete.
   [T]
7. It is possible to have $\chi > \omega$.
   [T: See the example in the text]
8. $\chi \leq 1 + \max_i \min\{d_i, i - 1\}$, where d_i is the degree of a vertex v_i.
   [T: Greedy Coloring with a better sequence gives this bound]
9. The fact that the Greedy Coloring does not always give optimum coloring is due to vertex sequence used in the Greedy Coloring rather than the Greedy Coloring itself.
   [T: See the text]
10. For an interval graph $\chi = \omega$.
    [T: Greedy Coloring gives optimum coloring for interval graphs]
11. Interval graphs are perfect.
    [T: A subset of intervals is another set of intervals]
12. A sub graph of an interval graph is also an interval graph.
    [T: Trivially true]
13. For any graph $\chi \neq \chi'$.
    [F: For a triangle, $\chi = \chi' = 3$]
14. Like in vertex coloring, a Greedy Coloring for edge coloring of a graph gives an upper bound of $\chi' \leq \Delta + 1$.
    [F: Gives an upper bound of $\chi' \leq 2\Delta - 1$]
15. $\chi' \leq \Delta + 1$.
    [T: This is the Vizing theorem]
16. For a bipartite graph, $\chi' = \Delta$.
    [T: This is the Konig’s theorem]
17. It is possible that $\chi' = \chi$.
    [T: For a complete graph $\chi' = \chi = \Delta + 1$]
18. Every planar graph is 5-colorable.
    [T: Since it is 4-colorable]
19. Every planar graph is 4-colorable.
    [T: 4-color theorem]
20. No planar graph is k-colorable, for $k \leq 3$.
    [F: A cycle is 2- or 3-colorable]